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THE PRODUCTION FUNCTION OF STUDENTS' GRADES

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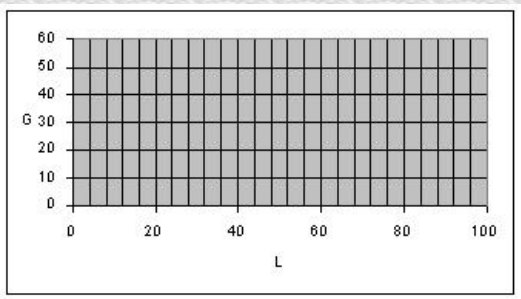
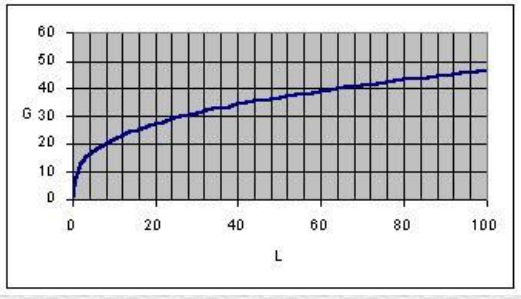
Introduction: the textbook starting point

The **production function** is often considered by undergraduate students as a difficult concept and its far-reaching consequences are not always well understood. To help students understanding the concept and its limitations, we propose to take into consideration a particular production process that should be well known to them: the production of a specific (higher or lower) grade in an exams after an education course. For instance, an ECON 101 at University.

Let's measure G, the **Grade** attained, on the Y (vertical) axis of a Cartesian space. Since **Labour** (the hours spent on books and attending lessons) is the main input, let's measure it (L) on the X (horizontal) axis.



The neoclassical approach usually states the following shape of the relationship, obtained through this dataset:



The higher the labour, the higher the marks. The productivity of first hours is very high (the grade sharply increases after the very first hours) while decreasing with time (the latest hours increase the grade only slightly).

Some comments

Although at first sight this approach may seem straightforward, some observations should be introduced.

Firstly, most of the points in that curve are not empirically measured but rather **notional**, i.e. based on a **what-if** statements (what grade would you take, **if** you went now to the exam), since actual examination grades will be actually obtained in one session only (or more, in the case the student does not succeed). You are not continouosly getting marks as long as you study, but just at the end.

Secondly, you may have **expectations** about which would be the grade, depending on hours spent on books, but these are subjective and you can't be sure that they are perfect. Some would say that they have to be **rational expectations**, so that when making several forecasts for different L, the sum of the mistakes ought to be zero (by taking negative mistakes if the grade is actually lower than the forecast and positive numbers in the opposite case). Our guess, however, is that there is no compelling reason why real students' expectations should fulfill so strict requirements.

Thirdly, the neoclassical basic textbook production function is deterministic and **associates bi-univocally one L with one grade**, with no stochasticity.

The student making a choice should know this relationship and treat it as objectively given, so that his choices will build on it and based on prices only, as we shall see in the next section.

Let us now **concede on all these three points**, by **assuming that there is indeed a 1:1 deterministic relationship between L and grade**, so to look at the **shape** of the production function as it is proposed.

Is it in your experience that the productivity of first hours is very high (the grade sharply increases after the very first hours) while decreasing with time (the latest hours increase the grade only slightly)?

An alternative view would point at three objections.

1. **Early hours are simply not productive**, in terms of final grades, since they are used just to orient oneselves in the most basic concepts. No questions on these concepts will appear in the final test - because they then would appear too easy - but it is a necessary pre-condition for you to go into the more difficult material, so to be eventually able to autonomously solve drills and questions.

Until you've read a large part of the assigned courseware, your grades would be zero (or at the minimum allowed) with no improvements for a while. **The production function would be flat, at the beginning**, at minimum marks.

2. What happens just before examination? Is then student work only very weakly productive? On the contrary, many student could claim that **last days are key to the final results**.

A **steep increase in productivity** is typical of **last days of study**. This is due to two effects: the intensity (number of hours per day) usually rise but, more importantly to the point, **each hour is particularly productive** to fix potential sources of mistakes.

Once understood, the lessons can now be repeated with fluency and give rise to unfragmented argumentations as well as autonomous thinking. The last repetition is extremely important to avoid minor and major mistakes in the exam, thus significantly changing the grades.

3. The proposed shape of the production function raises a further inconsistency with actual "production" of grades. The neoclassical production function is **unlimited** with respect to G. There is no maximum grade. At the exams, the student can get, if properly prepared, an arbitrarily high G.

In the real world, by contrast, in any exam, there is a theoretical maximum (eg. 100/100 with honours), beyond which there is no higher mark. **For high L, the function is flat, not rising**.

Even more: many students would argue that the theoretical maximum is unachievable and that there is an "institutional max",

because of the difficulty of the exam, its duration, and the rules of punishments for even minor mistakes. As a student could put it: "With such a professor, already a 80/100 is a miracle. Even if I studied twice as much hours, my grades simply cannot go beyond that".

The production function at work

Coming back to neoclassical shape of production function, it is commonly used to establish which is the optimal L given:

A. the monetary cost of L
B. the monetary revenue of G

For a discussion about how to obtain these two values see here [1].

Using these two data and the production function, a profit-maximizer producer (i.e. student) would choose a uniquely identified level of L, thus of grade [2].

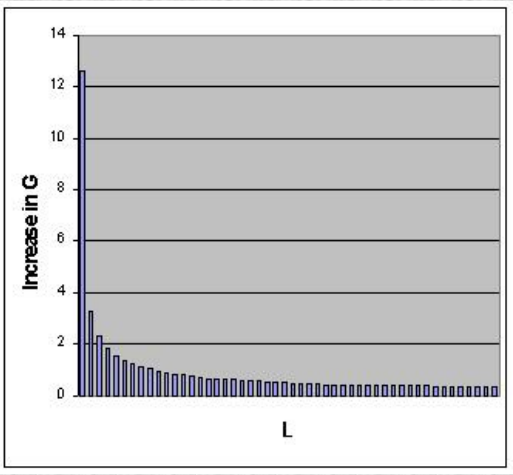
Let's see which is the optimal L level (L*) chosen by the neoclassical producer. This L* will be **the unique** to satisfy the equation:

Marginal productivity of L = wage

Three issues arise here:
1. What is marginal productivity?
2. Why is the profit maximised when that equation holds?
3. Which are the main factors influencing L*?

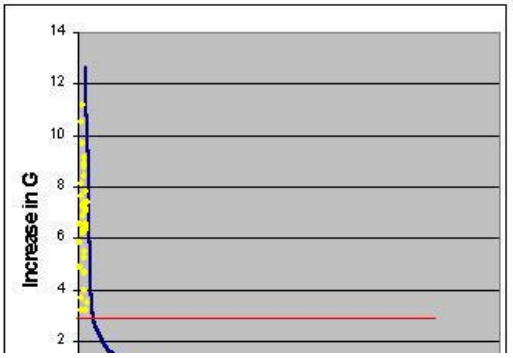
As for the first issue, marginal productivity of L is simply **the increase of grade** due to the **increase of one hour** of study, as you can see computed here in the fourth column D.

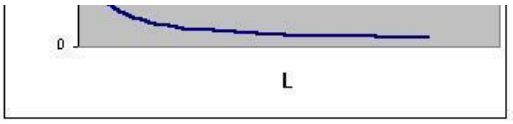
In the deterministic setting assumed by neoclassical theory, this is defined at every L level.



And it falls all the time. As you can see, the marginal productivity of L is extremely high at the beginning, then it continuously fall without never becoming zero or negative.

Given the fall in productivity, it is fairly clear why the point at which wage is equal to the labour productivity maximises profits, i.e. the difference between revenues and costs:





Be the red line the constant wage leve (between 2 and 4). Profits are the (yellow dotted) area between the labour productivity curve and the wage curve. They are positive until the optimal L*, when the (falling) labour productivity is exactly equal to the wage.

A larger L would have a cost (wage) higher than productivity, thus it implies a loss that would reduce the previously cumulated profits.

This setting gives straightforward answers to several questions.

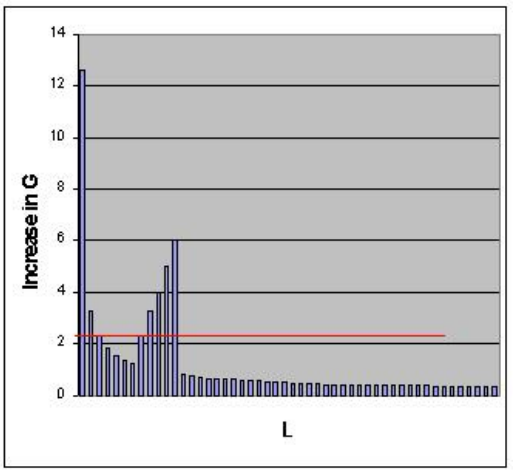
An increase of the wage (higher red horizontal line) reduces the optimal L*, thus the grade. The higher the wage, the lower the production. What to do to increase the production (and employment)? Just reduce the wage, they say.

For any given wage, a shift of the marginal productivity curve upwards has the opposite effect: longer hours of studying and better marks.

Some further comments

Different shapes

All this requires the assumed shape of the productivity curve, which in turn is derived from the shape of the production curve. If the productivity had a different path, one could not easily establish the optimal point through the abovementioned equation:



In cases like this, the rule based on smooth fall of labour productivity might not provide the optimal L.

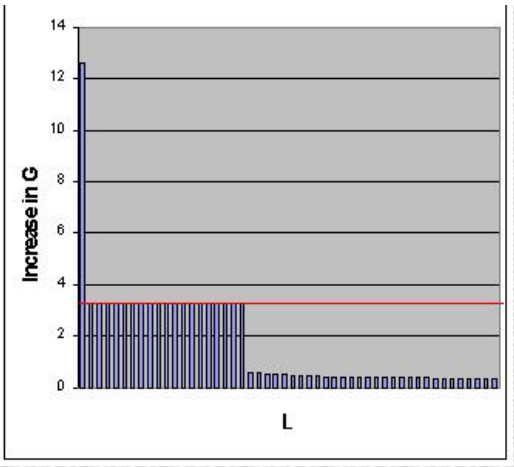
In other terms, the fall in labour productivity is a necessity for the standard neoclassical theory to single out optimal outcomes, irrespective of empirical experiences and objections.

Continuity

The mathematical condition of "continuity" for the productivity fall is a logical necessary condition to achieve a perfet 1:1 mapping between productivity and the L level. Given any productivity, there should be always one and just one L level for which it holds. Each hour of study has here a different productivity. If I tell you "an increase in your grade of 0.12232" you can answer: this is the productivity of the 67th hour of study.

The level of precision here implied is quite high and seems to exceed what it currently available in the consciousness of the decision maker. Can you really know this information? And is it true that each hour of your study has a different productivity? Aren't hours of study that have exactly the same productivity (or even have no individual impact on grade)?

This would make the crossing condition of equal wage and marginal productivity simply undetermined, since the condition holds for several L values:



It's not only an empirical problem; it's also a matter of principle: grades cannot be irrational numbers and the number of possible grades is finite. For instance, in a 0-10 scale there are only 11 possible discrete grades (or 21, if half a grade is allowed).

This reduces the number of marginal productivity values (a jump from a grade of 4 to 6 would mean a productivity of 2, the same as from 6 to 8). Indeed, the number of increments due to an addition of L is very limited.

In other words, it is not possible that given a marginal productivity you can "always" uniquely identify a L level, since the L levels are more numerous than the possible marginal productivities [3].

Quantity vs. quality

An interesting reinterpretation of the scheme of the "student production function" is to consider its time (L) as Research &Development efforts and the achieved grades (G) as the vertical quality levels of the differentiated service "exam".

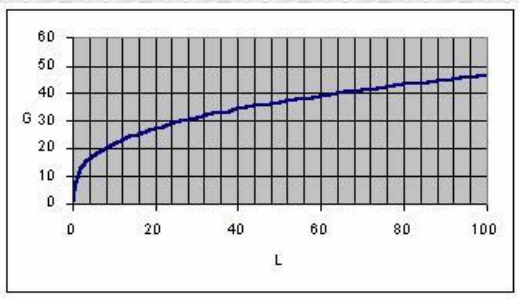
Instead of considering the G as quantities, one takes it as quality levels. We have widely discussed the question of product differentiation here, and produced a model with quality-based competition here, in which R&D output is highly stochastic.

Capital, technology, and student heterogeneity

Until now, we considered the influence on G only of L. However, a professor could be particularly good at explaining, another might be worse. Clear and in-depth books can greatly boost the productivity of L. In both cases, each hour is more effectively spent with a good professor or before a good book.

The neoclassical interpretation of this is twofold. Books are considered "capital" that complements labour in the production of grades.

The "better professor" case is instead intepreted as a "technology shift". In both cases, the production function moves upwards, in this way:



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For the same L, the student gets higher grades.

Neoclassical approach can thus treat and include in explanations these two factors.

It is far less successful in taking into account the individual ability of the student. His personal history, his innate talent, his overall intelligence, the way he organizes its time, his method of studying, the privileged relationships with teachers, his previous experiences in the field, his social background; all this tends to be left out of the picture.

In fact, the neoclassical assume, to prepare the field of their key concept of "perfect competition", that all firms have exactly the same production function.

If a firms has a different one (say, a higher one), the others will costless and timely imitate it (instantaneous adoption of "best available technology") [4].

By contrast, the tradition of evolutionary economics (from which this author draws his reflections) has put **heterogeneous agents dealing with difficult tasks** at the centre of the stage, underlining that firm have idiosyncratic **competences**, i.e. qualities difficult to imitate because due to the whole of their business history, organization, and identity.

Make your own experiments

Which is the shape of your production function? Which is the relationship between effort and results?

Try to answer yourself to these questions. Then, ask a student you know to draw on paper the production function of his latest exam. Which shape does he produce?

In another experiment, try to answer to the following questions: How do you choose the amount of time to devote to study a certain subject with a final exam? Do you take a global decision before beginning? Don't you take several sequential decisions on short-term commitments (e.g. "today until dinner")? Don't you revise your choice (e.g. "It's too boring, I go out instead of the planned further 4 hours")?

If this is the case, you can understand why, quite similarly, **firms do not always choose a global (e.g. yearly) production** in advance but **take sequential partial decisions**, based on feedbacks (e.g. cumulating inventories), negotiated between different organisation levels and sub-cultures (e.g. with some workers slowing down the rythm or with the quality controllers forcing re-processing of low-quality items).

Synthesis

The production function is a nice formal tool to introduce basic relationships in a production process but it is far too straight, deterministic and simplified to significantly interpret what happens in real firms.

The optimising rule about the production level requires assumptions which are not always met, while being quite close to alternative shapes coming out from empirical experiments and surveys.

Evolutionary economics, with its emphasis on agents' heterogeneity, routines, quality standards represent a promising alternatives to the neoclassical approach.

NOTES

[1]

A. In typical production processes, the cost of L is a hourly wage. But the application to the process at hand now is not so direct and obvious. Does the student receive dollars for every hour he studies? Since this is usually not the case, the neoclassical approach would point to the "opportunity cost" of studying, defined as the wage he is renouncing at while studying.

It should be unique, deterministic and objectively given. Well said. But in practice, how do you find it? How many jobs he could do instead of studying? A lot. But to know the exact wage he would get, one should have to ask the employer, because apart from very rough averages, the wage has an important firm-level component. And you

should repeat this procedure for all "possible" job, and find out the "highest" wage.

This is highly unpracticable. Accordingly, a direct monetary opportunity cost may be difficult to ascertain and to be used as a reference for further reasoning.

In another perspective, "wage" could be identified as the non-monetary opportunity cost that derives from the utility of activities the student is renouncing to, say go out to cinema.

In this way, wage is a cost due to a reduction in consumption.

Again, the problem of whether activities are uniquely identified arises. Not only that: when knowing that you'll be studying, probably you do not make a plan every two hours about the films to which you renounce. This indeed would increase the pain you feel. A strategy of downplaying the alternatives is usually used by students to avoid pain.

The so-defined "wage" is thus far from being "objectively" given to the decision-maker: it is a result of a conscious activity of the latter. The alternative cost is endogenous, not exogenous, in contrast to neoclassical assumptions.

B. In typical production processes, the revenue derives from selling the unit of product obtained. But it is very unlikely that somebody pays the student as soon as he succeed an exam, let alone in dependence to the grade.

The monetary revenue gained from an additional hour of study should then be computed in reference to its consequences in professional life. The marginal increase of the exam grade has an impact on final grade of the degree the student is taking (e.g. M.Sc.), which converts in higher probability of getting a higher-paid job, maybe even in influencing the time job stability and duration and histotal life career.

Sounds pretty nice but unreal, doesn't it? So much deterministic impact should have every small amount of studying!

In response to this objection, one could argue that studying is not a typical production process, in which, instead, the wage would be rather known, a simple good is produced and sold only once.

This argument reveals, however, that the neoclassical scheme is not very general. It might have troubles in explaining real production processes where things are more complex than in that standard case (e.g. in case of production by teams, informal knowledge accumulation with learning by doing, brand loyalty effects on re-purchasing of consumers, revenues based on royalties instead of "selling once and for all",...).

[2] No surprises at the exams: given a certain amount of hours spent on studying, the grade is automatic. This is the consequence of the assumption of a **deterministic** production function.

[3] See the two columns I and J here with integer numbers and you will see this effect.

[4] Some advanced neoclassical models can cope with some of the abovementioned elements.

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