Market share structures: an exhaustive list and a research agenda

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Abstract

With this paper, we release an exhaustive list of the 18 429 ways in which a market can be split across different firms. We provide a univocal nomenclature, compute a reference value of the Herfindahl index for each market share structure and lay down a research agenda for empirical and model-based studies, including evolutionary and history-friendly models. The list can be a valuable tool to compare models within and across different traditions of economic thought. While providing a first use of the list by classifying 98 269 real market share structures from an international database into the exhaustive list, we find clues that economics has devoted most of its attention (in this field) to structures which are almost negligible from an empirical point of view while addressing without enough precision the vast variety of actually occurring structures. In addition to several items for a renewed research agenda for empirical and theoretical industrial organisation, we set the stage for a comprehensive temporal morphogenesis of market structures in a dynamic perspective.

JEL codes: D40, L00, L10, L11, L13.

Keywords: Market structure, asymmetric oligopoly, imperfect monopoly, oligopoly core with a competitive fringe.

Introduction

In this paper, we carry out the unprecedented undertaking of producing a long, exhaustive, list of market share structures, containing every possible symmetry and asymmetry in the distribution of market shares across firms, by leveraging a recent mathematical advancement. For this purpose, we have to take a few assumptions and technical steps that guarantee the outcome. Once obtained the result, we highlight how it can be used to compare models and how it can give a contribution to the defragmentation of current economic debate.

Over the centuries, the economic theory has produced models capable of generating market structures of monopoly, oligopoly and (perfect or imperfect) competition, with homogeneous or differentiated products. The issue of the distribution of market shares has typically been embedded in broader considerations targeting the total market size, the prevailing price(s), the profitability of the firms[1]. An overriding consideration has been given to the sheer number *n* of firms operating on the market (Colander 2020; Ragan 2019; Kolmar 2017; Fine 2016). In the XIX Century, Cournot (1838) and Bertrand (1883) provided models of market share repartition when the product is homogeneous and competition is, respectively, on quantity or on prices, finding equilibrium solutions. The former ends up with competitors obtaining all the same market share, if all (two or more) firms face the same cost structures. The latter leaves undetermined the repartition of the market.

[1] As such, broad syntheses of the history of economic thought may even completely lack the words "market share" (as it happen with Faccarello and Kurz 2016 and with Hunt and Lautzenheiser 2011 and, in practice, as well for Barnett 2015, in which the sole quote of the words is related to an empirical example and not to theory, or for Sandelin 2014, where the sole quote is referred to mercantilism). An exception is Screpanti and Zamagni 2005.

The neoclassical tradition, as percolated in contemporary textbooks, has expressed a preference for symmetric conditions, which divide the market equally across the *n* firms (perfect duopoly, symmetric oligopoly, perfect competition with equal shares) (Polo 1993; Besanko & Braeutigam 2011; Krugman & Wells 2018, Perloff 2018; Acemoğlu 2019; Mankiw et al. 2020; Vohra 2020).

Perfect competition has been considered originating from a plethora of identical firms, ending up with the same market share. For instance, Mankiw et al. (2020) introduces the concept with an example as this: "Consider a market with 1000 identical firms… because the firms are identical, the quantity supplied to the market is 1000 times the quantity supplied by each firm" (p. 345).

One of the few historical exceptions**,** the Stackelberg model (1934), divides the market in unequal share due to an assumed asymmetry in timing of the firms' decisions. With a linear demand curve, the first mover obtains double the market share than the follower (Perloff 2018, p. 528). Modern extensions of Cournot and Bertrand to differentiated products can produce unequal market shares, although no exhaustive mapping is carried out (e.g. Perloff 2018, p. 540).

With horizontally differentiated products, the Hotelling model (1929) distributes the market in two equal parts, if the two companies have the same costs and the "distance" that the consumer needs to "travel" along the differentiation axis (which can be interpret in a geometrical or non-geometrical way) is the same in all directions (Vohra 2020, p. 104-108).

In the chapter on imperfect and monopolistic competition of a typical neoclassical textbook (Mankiw et al. 2020, p. 355-372), no reference is made to the exact type of variety in companies' costs and market share, leaving the entire discussion on market shares at a verbal informal level. Vagueness is even considered as inevitable ("Because reality is never as clear-cut as theory, at times you may find it hard to decide what structure best describes a market. There is, for instance, no magic number that separates 'few' from 'many' when counting the number of firms", Mankiw et al. 2020, p. 357).

More complex models, such as Dixit (1980), Schmalensee (1981), Vives (1985), Maskin (1986) can lead to differentiated market share, although discussions tend to concentrate rather on prices, quantities and profits and to neglect explicit market shares (e.g. Tirole 1988, p. 314-323).

Game theoretical approaches have modernized the tools of the discussion. In the words of Kóczy 2018: "Now industrial economics (or industrial organisation), the study of interaction of firms in markets, is unimaginable without game theory. Depending on the number of interacting firms (one, two, or more), we may talk about monopoly, duopoly, or oligopoly, but we use oligopoly as the generic term of these models. Depending on the choice variables we may talk about a Bertrand oligopoly, a price competition (Bertrand, 1883), or a Cournot oligopoly, a quantity competition (Cournot, 1838) that is really a competition in capacities. For a general introduction on these models, see any textbook on industrial organisation, such as the book by Tirole (1988) or Cabral (2000)". As you see, after boasting the relevance of game theory, the issue of market share structures is quickly converted into an issue of the number of firms and the examples given are sheer reinterpretations of XIX Century models. Indeed, in Cabral (2000), notwithstanding the fact that the topics covered in the book include product differentiation, advertising, mergers and acquisitions, research and development, networks, standards and path dependency, the only models quoted in the chapters' title remain Cournot and Bertrand. Tirole (1988) contains a plethora of game-theoretical models of competition, usually described in one or two pages, but reserves as many as 34 pages to Bertrand and 35 to Cournot, in their classical shape or in more advanced variations.

Since we shall not only provide plenty of market share structures but shall provide a method to empirically detect them over real time, it's important to underline that game theoretical models, although they have explicitly introduced multi-stages games, do not usually relate each stage to a different period of actual historical time. The classical Tirole (1988) uses more than 36 times the word "dynamic" or "dynamics" but does not contain any single example in which two stages are separated by actual time (e.g. with Stage One corresponding to a certain year and Stage Two to the following year)[2]. It includes plenty of multi-stage and repeated games, with deliberately temporal verbal expression such as "lags", "quick", "speed of adjustment". It covers games where time is object of active choice (e.g. the durability of the produced good, the time to entry, the time race to patent). Nevertheless, the logical time of decision-making is not paired with a "chronometric" time expressed in units such as months and years[3].

Also the "New empirical industrial organization" (NEIO), which has a keen interest in the empirical investigation as we do, is characterised by this uncertain relationship with the real time. In the words of Roy et al. 2006: "The repeated nature of the game probably leads to this blurring of the definition of ‘game-theoretic reaction’and ‘reaction over time'".

Recent models (such as Buccella 2015; Yamane 2018; Toshimitsu 2021) continue to draw on Cournot, Bertrand or other classical models, extending them in highly advanced frontiers (such as product differentiation, network externalities, profit-raising entry, workers' unionization, mixed oligopoly where profit-maximizing private firms compete against welfare-improving public firms), with impacts on the market shares that, although allowed to be different across firms, are not systematically investigated.

A specific strand of research has been operating on market share to explore the issue of concentration, with historical roots in the work e.g. of Bain (1956), and recent extensions to macro-dynamics and politics (Baker 2017; Van Reenen 2018; Stiglitz 2019; Syverson 2019). In this case, concentration is summarised with a single indicator and the wealth of market share structures is sidestepped in favour of econometric tractability in connection with other variables.

We obviously cannot account to every paper using market share as relevant variable in this inevitably coarse literature scan[4]. Nonetheless, we cannot but notice that this rich debate in the scholarly journals is contrasted by very little uptake by textbook and actually a diverging trajectory: several ones are nowadays stripping away the names of the models' authors reporting not a single quote of Cournot, Bertrand or Stackelberg (O'Sullivan, Sheffrin and Perez 2017; McConnell 2018; Tucker 2018; Hubbard 2019; Ragan 2019).

Outside the neoclassical paradigm, there has been a much more pronounced attention to asymmetric oligopoly, notably with Sylos Labini (1967), which fully recognises that oligopoly can be asymmetric and can coexist with small firms. This analysis has been extended by Sylos Labini (1992) and positively emphasised by D'Alessandro et al. (2017). Also Hines (1957) underlines that many

[2] The only exception is a narrative talk aimed to distinguish fixed costs from sunk costs (p. 307-308, Tirole 1988). Conversely, in the appendix on game theory, the distinction between static and dynamic games is reduced to an issue of information (p. 424, *idem*).

[3] For a constructive discussion of the difference between logical and chronometric time, see Piana (2019).

[4] For instance in narrower industry studies, an oligopolistic core is complemented by a competitive fringe in Devine and Siddiqui (2020).

markets show up a competitive fringe and an oligopolistic or monopolistic core. He explores two consequences of this view for the entry process, by distinguishing two "strategic groups". Completely new firms usually enter at a small scale, and remain locked in the competitive fringe. By contrast, established-firm entrants might move at once in the core. In other terms, the number *n* of firms is endogenous and there is no assumption that the entrants will get the same share as the incumbents. Conversely, in recent innovative approaches reframing microeconomics in a broader civic context, inclusive of sustainability and well-being, there have been definitions of oligopoly as a market structure in which several companies operate, but none of them has a negligible market share (Becchetti et al. 2020, p. 168).

Evolutionary economics, which we consider as an alternative paradigm to neoclassical economics, challenging not only the assumptions but also the method and the criteria for evaluation of models, has produced a yet richer picture of industrial dynamics (Nelson and Winter 1982; Dosi, Malerba and Orsenigo 1994; Dosi and Nelson 2010; Klepper and Malerba 2010; Nelson et al. 2018). With its replicator dynamics (Silverberg, Dosi, Orsenigo 1988; Saviotti and Mani 1995; Safarzynska and van den Bergh 2011; Holm, Andersen and Metcalfe 2016; Dosi, Pereira and Virgillito, 2017) and Polya Urns (Arthur et al. 1983; Witt 1997; Dosi, Moneta and Stepanova 2019), it has been generating a vast variety of market share structures in temporal sequence.

In particular, history-friendly models (Malerba, Nelson, Orsenigo, Winter 1999; Orsenigo 2005; Yoon and Lee 2009; Malerba, Nelson, Orsenigo and Winter 2016; Brenner and Murmann 2017; Capone, Malerba, Nelson, Orsenigo, Winter 2019) allow not only to mimic actual business histories by capturing key elements of the appreciative narratives used to interpret them but also to generate counterfactual simulation runs about how differently market shares (and other key elements of market structure) could have evolved. They explicitly map the simulated periods into historical time (months, years).

Evolutionary models of industrial dynamics extends the analysis from the single market to submarkets (Klepper and Thomson 2006) and across several markets (Wegberg 1993; Vonortas 2000). They clearly recognize the importance of markets where an oligopolistic core is complemented by a competitive fringe, sometimes small, sometimes large (Garavaglia, Malerba, Orsenigo & Pezzoni 2014) and of turbulence in market shares (Dosi, Marsili, Orsenigo and Salvatore 1995). In some models, market share dynamics has been coupled with price routines (Bloch and Metcalfe 2018; Almudi, Fatas-Villafranca, Palacio and Sanchez-Choliz 2019). A strand of research has reframed the issue of market shares from a microeconomic to a macroeconomic (Seppecher, Salle & Lavoie 2018; Dosi, Napoletano, Roventini and Treibich 2019) and international level (Dosi, Roventini and Russo 2019). The dynamics of market shares has been considered as a contributor to labour market trends, including de-unionization, and to their macroeconomic consequences (Dosi et al. 2020)[5]. But even if this tradition of thought has widely extended the number of market structures of interest, it has hesitated to establish a full plan of investigation, probably because of the feeling that the number of market structures may turn out to be unlimited, thus inexhaustible to a systematic and comprehensive treatment.

[5] Beyond economics, market shares have been an obvious object of interest for the business disciplines and for empirical investigations, including for instance the seminal book by Cooper & Nakanishi (1989) and the on-going data collection called "Profit impact of marketing strategy", utilized by Buzzell & Wiersema (1981) and by Edeling & Himme (2018). It would be impossible to quote now all – or even a significant part – of this production in business disciplines, including marketing. We simply note that our proposal will offer to such disciplines and investigations an additional tool of interpretation.

In this paper, we take a few decisions that make sensible and feasible to close down an exhaustive list of all possible market share structures, building on a few previous mathematical and abstract works, and we give a name to each of them (a simple identification acronym). In this current undertaking, we limit ourselves to the descriptive side, whereas almost all the literature has considered the distribution of market shares something to be embedded in reasons, causal explanations, and normative arguments of desirability. The literature has mostly taken a reductionist approach to market share structure, on the one hand, focusing on the sheer number of firms operating on the market and, on the other hand, quickly connecting market share with market power, including with the issue of price-making or price-taking behaviour. The main normative discussion leveraging the analytical tools related to market shares has been around the anti-monopolistic and anti-trust legislation, for which a key issue is determining the definition of the market itself and its size[6].

The same vocabulary used (e.g. monopolist) is not neutrally saying that a company has the 100% of the market but immediately takes a judgemental tone ("to monopolise a conversation" is typically not of good manner). Since in our methodological approach we stand for researchers to openly embrace value judgement, so that their advice can be received by specific agents-of-change in society and politics, we are particularly keen to set a very broad general background for description upon which, in a separate manner, value judgements can be rooted in a variety of explicit criteria. Our rejection of naive positivism and instrumentalism (Friedman 1953) hinges on the adoption of pluralism (Piana 2020), whose first step is to generate the vastest possible variety and whose second step is to organize it (e.g. in clusters). In a way, we invert the classical Baconian sequence of "pars destruens, pars construens". Accordingly, this paper (which accompanies the release to the community of statisticians, business analysts and historians, model builders and teachers an exhaustive, thus in a sense final, list of market structures) is also called to provide a positive agenda for the next steps.

The paper is organised as follows: in the first part, we take an abstract road and we refer to the theorem and the Java code that allows us to generate the exhaustive list, after having set the restrictions for its establishment. We conclude the first part by laying down an extensive research agenda for abstract processes to generate the temporal morphogenesis of the market share structures and for models to be compared for their capability of generating some, most, or all the items of the list. We aim to foster cumulativeness and repartitions of tasks across models of the same conceptual family, especially for evolutionary economics but also for other heterodox strands. We highlight the need for a partition of the exhaustive list that would allow a new human-friendly descriptive vocabulary for market share structures.

The second part turns empirical and a very broad catalogue of real-world market share structures is utilised to "populate" the list with frequency distribution. The catalogue is imperfect in several respects but since its size is several times larger than the already long list of market share structures, the lack of empirical evidence for certain ones becomes highly significant. The finding that symmetrical distributions of market shares cover only the 0.06% of empirically detected structures within the database provides ground for the testable empirical thesis that the economic science has been devoted most of its efforts in the field of market structures for situations that are empirically

[6] "[…] gauging real competition in the twenty-first century marketplace—especially in the case of online platforms—requires analyzing the underlying structure and dynamics of markets" (Khan, 2016). The author has been nominated chairperson of the Federal Trade Commission in June 2021.

negligible. Conversely, for the interested researcher, this database, covering 27 years, 128 countries and 26 industrial sectors (covering a significant share of global GDP) provides a starting point for transition matrixes (from one structure to another) and an investigation about the stability of structures, in terms of persistence and resilience. We thus detail a second round of research agenda proposals. In particular, we signal further databases of relevant data that might provide a better field for empirical distributions.

Part I

Definitions and assumptions

The market share is the part of a market covered by a firm, in percentage terms[7]. The sum of all firms' market shares is 100%. The distribution of market shares across all firms is the "market share structure" (from now on: MSS).

In order to obtain an exhaustive enumeration, we shall need to make certain additional assumptions, while discarding others. The company turnover in absolute monetary values (the firm size), object of many empirical and theoretical studies (e.g. Axtell 2001; Fujiwara et al. 2004; Zhang et al. 2009; Delli Gatti et al. 2008), should be translated into a relative value of share of a total. If not, one would have an open-ended large value.

Once avoided the unlimitedly large, we need not to fall in the trap of the infinitely small: the percentage of the share cannot be defined with an infinite number of decimals. On the contrary, one needs to quantise the percentage of the share. In particular, for the list we release, we quantise the percentage to the single unit of percentage (e.g. 32%, 47%, etc.). When detecting the structure empirically, this will require rounding, for which we adopt the banking convention[8].

In order to have an exhaustive list of all possible market share structures, we shall need to undertake a certain transformation of its simplest notion, in which labels of firms are accompanied by their own market share.

Table 1 – The basic way to represent the distribution of market shares

Company name	Market share
L_1	MS_1
L_2	MS_2
L_3	MS_3
…	…
L_n	MS_n

[7] Depending on the definition of the market, firms can be retailers or producers. In the latter's case, "[a] producer’s market share is the fraction of the total industry output accounted for by that producer’s output" (Krugman, 2018).

[8] According to the banking convention, one rounds to the nearest number. If the value is equidistant from two integers, the even integer is chosen.

The number of companies active on the market can rise to very large value. If the number of companies were allow to play a role in the definition, we could not close the set. Accordingly, we need to discard the most basic option, represented in Table 1, and take instead an original road, whose initial restrictions will be more than remunerated with the great length of the path it opens.

The method

We define a "market share structure" as a row vector collecting 10 elements, each defined as follows:

$$\left\{ \begin{array}{c} x_i = \sum_{MS_j \geq Tmin_i}^{MS_j < Tmax_i} MS_j \\ Tmax_i = 10 \text{ x } (11 - i);\ Tmin_i = 10 \text{ x } (10 - i) \end{array} \middle| i = 2, \ldots, 10 \right\} \quad (1)$$

$$\left\{ \begin{array}{c} x_i = \sum_{MS_j \geq Tmin_i}^{MS_j \leq Tmax_i} MS_j \\ Tmax_i = 10 \text{ x } (11 - i);\ Tmin_i = 10 \text{ x } (10 - i) \end{array} \middle| i = 1 \right\} \quad (2)$$

where *i* is an integer in the interval [1-10], MS_j stands for the market share of firm *j*, and the sum extends to those MS_j whose value is equal or higher than the minimum threshold $Tmin_i$ and lower that the maximum threshold $Tmax_i$ (with the exception of i = 1, where the interval includes also $Tmax_1$).

We collect the x_i variables in a table, released with Appendix A, thus in rest of the paper we shall be calling the members of the vector as "columns" interchangeably. In the first variable x_1 (that we shall report in the first column of the table of the exhaustive list in Appendix A), one writes the value of the sum of the market share of companies having individually a market share of 90% or more. Evidently, there may be only one of such a companies (two companies cannot have each 90% or more). Thus, applying formula and reminding the quantisation, the domain of the first variable will only include the following values: [0, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100]. The zero characterises the cases where no company has a market share of 90% or more.

The second element of the vector (second column) will contain the sum of the market share of the companies having each a share equal or higher than 80% lower than 90% (and again there will be only one or none). Because of the rounding convention taken above[9], the domain of the second variable will include the following values: [0, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90].

The fifth variable (column) will contain the sum of the market shares of the companies having each a market share included in the interval [50%, 60%). Here, for the first time, two companies can belong to the interval, so the domain will be [0, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 100]. The case in which the fifth column is 100 represents the (symmetric) perfect duopoly. In the last column, you have the sum of the market share belonging to companies that individually have a share of more than 0% and less than 10%. This column can take any value [0-100]. A 100 in the final column means

[9] The value 90 is obtained when the true share is something like 89.8%, so rounding to the nearest number gives 90. If the company has exactly 90%, it does not belong to this column but to the previous one.

that all companies are "small". The classical perfect competition would generate market structures contained in such a vector[10].

Not all combinations of individually legitimate values of the columns sums to 100. This is very positive, since the number of all combinations is 2.59272E+14, which would provide a too large a catalogue of market share structures.

In order to obtain the full list of all possible values that these 10 variables can take in a consistent manner, we apply the theorem developed by Page (2012), which guarantees that all and only the ordered values who distribute 100 in legitimate way are enlisted. It's a fairly complex theorem, which Page has trasformed into Java code for actual algorithmic execution. The amazing flexibility for economic purposes of Page's algorithm has been highlighted by Piana et al. 2020. Indeed, the present paper is built upon and extends the chapter 14.2 of the latter.

Results

Taken the assumptions and applied the method, we obtain the exhaustive list of the 18 429 market share structures, which contains all and only the legitimate values of the ordered 10 variables whose sum is 100. This list is closed and final, in the sense that in any empirical or model-based study you cannot find in any market a structure that, appropriately transformed, is not contained in such a list. It contains every possible symmetry and asymmetry in the distribution of market shares.

By applying the naming convention by Piana et al. 2020, we order the list in descending order of the first variable, then of the second, the third, etc. and we add a prefix beginning with ID to remind that the alphanumeric code is a unique identifier. Thus, we set IDMSS00001 as the name of the first market structure, the perfect monopoly with one firm covering the 100% of the market. The name of the last market structure is IDMSS18429 and corresponds to the case where all firms have a small market share, which would be normally considered as highly (or even perfectly) competitive[11]. The perfect duopoly is IDMSS01119. In IDMSS04236, one company has the 49% of the market, the 47% of market is occupied by firms having between 10% and 20% of the market, 4% is distributed across one or more firms having less that 10%[12]. You find this list in Appendix A.

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[10] When, as in Part II, we detect which MSS represents a given distribution of market shares for which we have the individual firms' share, we sum according to the formulas. In case the sum of the market shares is not an integer percentage, we round the sum according to the banking convention: we round to nearest integer; if two integers are equidistant from the sum, we round to the even number. For example: 4.34% is rounded down to 4%, 4.65 is rounded up to 5%, 4.5% is rounded to 4% and 5.5% is rounded to 6%. In most cases, this assures that the sum of the members of the row vector (columns) is 100%. For the few exceptions (which lead the sum to be 101% or 99%), the 1% discrepancy is imputed to the largest value among the 10 columns, resulting in the lowest relative modification. If there are two equally high shares at the top, the rounding (up or down) will be for the leftmost column. Please note that the rounding is performed on the sum, not on the market share of the single firm. This means that, in the abovementioned example of 1000 firms with equal shares of 0.1% each, their sum is 100 and no rounding is necessary to attribute 100 to x_{10}.

[11] Following up this reference to the issue of market concentration, in the Appendix C, we discuss the application of the Herfindahl index to the market structures and release the corresponding values for all of them.

[12] A unique number of companies operating on the market is not determined, since the structure is compatible with a few different situations.

Please note that the list is the same for market shares in physical units (e.g. number of cars) and in value (e.g. the total revenue made by selling many different cars at personalised prices or the total revenue made by multiplying the physical unit of an homogeneous good with its unified price, if such price exists).

Research agenda

The exhaustive list is extremely useful to judge and compare a large set of models of industrial economics. You take a model which draw on a tradition (e.g. Cournot; Stackelberg; Malerba, Nelson, Orsenigo, Winter 1999) and you check whether it is capable of generating the full list of IDMSS or a part of it. Most likely, in the case of equilibrium models, you will start from the final equilibrium equation, discretise it and perform a systematic variation of its parametres (using a certain step for its discretisation), for a vast array of possible values. Then you attribute the market shares that it generated to the list and check whether a certain IDMSS has occurred. One could visualise the outcome of this procedure, applied to several models stemming from several traditions of thought, as exemplified in Table 2:

Table 2 – Fictitious example of systematic comparison of models
as for their capacity of generating MSS,
with only two models per tradition of thought and only two traditions

	Neoclassical models		Evolutionary models		N. neoclassical models capable of generating the MSS	N. evolutionary models capable of generating the MSS
	Cournot	…	Malerba et al. 1999	…		
IDMSS000022	Yes	No	Yes	Yes	XX	YY
IDMSS001023	No	Yes	No	Yes	XXX	YYY
Total number of MSS which the model is capable of generating	1	1	1	2		

It will be very interesting to see which IDMSS can be generated by all models and which only by some. If a family of models is totally incapable of generating it whereas another family is capable, this is an interesting new argument in favour of the latter[13].

Perhaps there exist IDMSS that cannot be generated by any current model – thus prompting for new models. This would be particularly important if such IDMSS has an empirical evidence, as we shall be exploring in Part II[14].

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[13] We are here proposing to extend to models of different tradition of thought what Tirole (1988, p. 262) calls "comparative modeling", indicating that the questions it poses "should be given high priority on the research agenda".

[14] This way of proceeding has something in common with that in NEIO is called the Menu Approach, also referred to as the Non-Nested Model Comparisons (NNMC) approach. "This method requires the alternative

In a second way to leverage the exhaustive list, building on the previous matrix, every model capable of generating an IDMSS would provide the reasons that justify the latter. For instance, a Cournot model would probably point at difference in cost structures among competitors to justify asymmetries in their market shares, as expressed by certain IDMSS. Thus, the model would indicate to empirical researchers having identified the IDMSS in the real-world what to look at. It may well happen that they would confirm or reject the reasons the model is suggesting, based on empirical assessment of them. For instance, it's possible that in order to obtain certain IDMSSS, a Cournot model would require extreme differences in the cost structures that an empirical analysis might not find. Accordingly, the falsification potential of the IDMSSS tool should not be underestimated (Popper 1959). More in general, explanations of IDMSSS are obviously called for, including for instance the recombination of knowledge during the industry life cycle, as "actors transform the knowledge into products and eventually in market shares" (Kalthaus 2020) or the role of demand structure and technological tegimes (Yu, Shi, Sadowski and Nomaler 2020).

A third way to leverage the exhaustive list to judge models is to check whether a model can endogenously generate a certain sequence of IDMSS without external shocks deriving from *ad-hoc* assumptions. This can highlight some advantages for certain families of models. For an agent-based economic model, it's fairly natural to generates endogenously sequences of IDMSS without any further change prompted by the external modeller. Most neoclassical equilibrium models, by contrast, tend to conclude by finding the equilibrium condition that cannot be modified without exogenous changes and shocks. For sequences of IDMSSS of particular importance, it would be worth investigating whether there are sequences of reasonable exogenous shocks that would induce the model to generate such sequence. In parallel, an important venue of investigation is about the abstract processes leading from one IDMSS to another, in line with the suggestions of the Ch. 7 of Piana et al. (2020).

In summary, we suggest comparing and judging models by pivoting the exhaustive list and we envisage the possibility of mapping areas (MSS) waiting for new models to justify them.

In another vein, the exhaustive list itself can be further complemented by recognising that is fairly long. It would be advisable to be able to produce a mathematical partition of it, so as to reduce the number of qualitative category, as envisaged in ch. 7 of Piana et al. (2020). By partitioning the list, you create non-overlapping subgroups of IDMSS whose union is the exhaustive list. It is not particularly difficult to produce a whatever partition, but what would be worth crafting with care and creativity is its correspondence to a new vocabulary aimed at describing the market share structures, without the burden of judgmental and behavioural additions. It's a big task, because it calls into question a long tradition of conceptualising markets. It should be noted that, once established such correspondence, the definition of each term of the new vocabulary would be referring to a closed list of IDMSS, thus would receive an unequivocal definition, that would have met the positive recognition by Wittgenstein's Tractatus logico-philosophicus (1921).

The partition would make much more manageable the study of the transition matrix from an IDMSS of a partition class to an IDMSS of another partition class. In absence of partition, the full transition

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competitive models to be developed and the solutions obtained under different assumptions about competing firms' behavior such as Nash, Stackelberg, etc. Assuming that the observed market data reflect the equilibrium corresponding to a particular mode of conduct (e.g. Nash or Stackelberg), the mode of conduct that provides best fit to the data is considered the most accurate description of the competitive structure of the market" (Roy et al. 2006).

matrix is somehow too extensive (18 429 x 18 429 = 339 628 041). A partition that would reduce the qualitatively different market share structures into 20 categories would have a transition matrix of only 400 cells. To keep track of more that one period back in time, 160 000 cells would draw on four periods, however defined, in the past. More in general, this machinery expresses its full potential when the sequence of IDMSSS is long and historically well rooted. In such case, it provides a neat formal tool for absorbing and interpreting the results of history-friendly models. It would also serve to demonstrate that, far from going back to a vision of history deprived of possible general explanations, such models allow for a fertile dialogue with true historians.

The list if particularly useful for models that rely on numerical methods, but it can be used also in numerical instantiations and parametrisation of analytical models. Given its exhaustive character, it can be used to map all possible total costs, profits, employment or other variables dependent on a given total market size and all possible market share structures.

More technically, in terms of concentration measures, one can compute the measure for each IDMSS and then establish a systematic relationship between couples of measures. For instance, in Appendix C you find the computation of the Herfindahl index; if you similarly compute C(5), the sum of the market shares of the largest five firms, you can establish the range of Herfindahl index corresponding to each value of C(5). Actually, what you find is not only the range, but the full distribution. In Appendix C you find such distribution for the Herfindahl index, which is far from being uniformely distributed.

Finally, it should be noted that the released list can refer not only to firms' market shares but also to anything that sums up to 100%. In industrial economics, it can be used in reference to employment and profits, investments and stocks, to technology competition (e.g. for the market share of road vehicles by alimentation), or to the use of limited inputs (e.g. land). In broader economic studies, it can for instance be applied to income distribution and regional distributions. Beyond economics, it can be used e.g. by political studies of electoral results and polls, opinion surveys, etc[15].

Part II

Data

In this part, we classify according to the above-mentioned exhaustive list of shares all the real structures detected in the CEPII Trade, Production and Bilateral Protection Database (henceforth: TradeProd), produced and released by Cepii (Centre d'études prospectives et d'informations internationales). It's a major dataset coupling international trade and domestic production by ISIC 3-level codes, covering 128 countries and ranging from 1980 to 2006[16]. It has been quoted more than 150 times from the moment of latest publication in 2012[17,18]. The database does not directly refer to

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[15] For clarity's sake, the researcher interested to apply the list to new fields may will want to change the acronym of the structures (the rows of the list in Appendix A).

[16] It can be downloaded for free after registration from
http://www.cepii.fr/CEPII/fr/bdd_modele/presentation.asp?id=5.
For description and first use see De Sousa et al. (2012).

[17] http://scholar.google.com/scholar?hl=it&as_sdt=2005&sciodt=0%2C5&cites=9456586636340758857&scipsc=&q=de+Sousa+J.%2C+Mayer%2C+T.+%26+Zignago%2C+S.+%282012%29+Market+Access+in+Global+and+Regional+Trade+Regional+Science+and+Urban+Economics&btnG=

[18] For a description of an earlier version, see Mayer et al. (2008).

companies and brands but we shall rather rely on the assumption made by Paul Armington (1969) that products traded internationally are differentiated by country of origin, which has become a standard assumption of international computable general equilibrium models. In other words, countries exports into a market are proxies for firms and we do include, differently from what would happen using trade-only datasets as (Cepii) BACI and (UN) Comtrade, the domestic production, consolidated from all local companies, as one firm. In the later discussion on the research agenda, we do suggest to look for other datasets, better corresponding to the strict market share definition. However, the key advantages of the dataset we are now going to analyse is its size and time coverage. Non-empty records, with an absolute total market size equal or larger than 1000 US dollars, are as many as 98 269, which means that a random distribution of markets to IDMSS would lead to an average of slighly more than 5 occurrence per MSS. The possible absence of occurrences for a certain IDMSS is thus not an in-built feature of the dataset size and can be considered as highly significant.

Method

We aligned trade imports and domestic production by country, ISIC code and year. We divided absolute value (expressed in dollars) by the total of import and domestic production. These shares are exactly the same that would accrue to domestic sales, if re-exports are computed based on the methodology in appendix B. This shares are added up in the ten categories of the IDMSS ($\Sigma(MS_j) | MS_j >= 90\%$; $\Sigma(MS_j) | 80\% <= MS_j < 90\%$; $\Sigma(MS_j) | 70\% <= MS_j < 80\%$; $\Sigma(MS_j) | 60\% <= MS_j < 70\%$; $\Sigma(MS_j) | 50\% <= MS_j < 60\%$; $\Sigma(MS_j) | 40\% <= MS_j < 50\%$; $\Sigma(MS_j) | 30\% <= MS_j < 40\%$; $\Sigma(MS_j) | 20\% <= MS_j < 30\%$; $\Sigma(MS_j) | 10\% <= MS_j < 20\%$; $\Sigma(MS_j) | 0\% <= MS_j < 10\%$) and the sum has been rounded according to the banking convention[19]. For each market, we determined the IDMSS (for a simple MS Excel file that allows you to do the same with your own data see Appendix D). Finally, we counted how many times (occurrences) every IDMSS has been detected in the real data (see Appendix B for full results).

Table 3 – The sequence of computations to identify the market share structure in one specific market. In green, you find one row of what we shall be calling in the rest of the paper the "MSS matrix". By repeating this procedure for many markets, you get the full "MSS matrix".

Company name	Market share	MS_j>=90%	80<= MS_j <90%	70%<= MS_j <80%	60%<= MS_j <70%	50%<= MS_j <60%	40%<= MS_j <50	30%<= MS_j <40%	20%<= MS_j <30%	10%<= MS_j <20%	0%<= MS_j <10%
L_1	MS_1		MS_1								
L_2	MS_2				MS_2						
L_3	MS_3				MS_3						
…	…										
L_n	MS_n										MS_n
	Σ		MS_1		MS_2+ MS_3						MS_n
	Round		round(s_1)		round(MS_2+ MS_3)						round(MS_n)
	Σ = 100	0	round(MS_n)± 1 (if necessary)	0	round(MS_2+ MS_3) ± 1 (if necessary)	0	0	0	0	0	round(MS_n) ± 1 (if necessary)
		IDMSS									

[19] We followed the procedure indicated in Footnote 9.

Results

Even if the database contains several times the total number of MSS of the exhaustive list, only 1 008 out of 18 429 actually have been empirically detected (the 6%). 94% of theoretically possible elements of the morphospace of market shares do not exist. The top 20 market share structures in terms of occurrences are in the following Table 4:

Table 4 – Top 20 MSS in the TradProd dataset

IDMSS	N. occurrencies	$\Sigma(MS_j)$ | MS_j >=90 %	$\Sigma(MS_j)$ | 80%<= MS_j <90%	$\Sigma(MS_j)$ | 70%<= MS_j <80%	$\Sigma(MS_j)$ | 60%<= MS_j <70%	$\Sigma(MS_j)$ | 50%<= MS_j <60%	$\Sigma(MS_j)$ | 40%<= MS_j <50%	$\Sigma(MS_j)$ | 30%<= MS_j <40%	$\Sigma(MS_j)$ | 20%<= MS_j <30%	$\Sigma(MS_j)$ | 10%<= MS_j <20%	$\Sigma(MS_j)$ | 0%<= MS_j <10%
IDMSS00001	80016	100	0	0	0	0	0	0	0	0	0
IDMSS00002	3471	99	0	0	0	0	0	0	0	0	1
IDMSS00003	1819	98	0	0	0	0	0	0	0	0	2
IDMSS00004	1234	97	0	0	0	0	0	0	0	0	3
IDMSS00005	943	96	0	0	0	0	0	0	0	0	4
IDMSS00006	823	95	0	0	0	0	0	0	0	0	5
IDMSS00007	614	94	0	0	0	0	0	0	0	0	6
IDMSS00008	540	93	0	0	0	0	0	0	0	0	7
IDMSS00009	476	92	0	0	0	0	0	0	0	0	8
IDMSS00010	464	91	0	0	0	0	0	0	0	0	9
IDMSS00015	258	0	89	0	0	0	0	0	0	11	0
IDMSS00018	228	0	88	0	0	0	0	0	0	12	0
IDMSS00027	221	0	86	0	0	0	0	0	0	14	0
IDMSS00022	212	0	87	0	0	0	0	0	0	13	0
IDMSS00012	188	90	0	0	0	0	0	0	0	0	10
IDMSS00033	184	0	85	0	0	0	0	0	0	15	0
IDMSS00040	179	0	84	0	0	0	0	0	0	16	0
IDMSS00048	166	0	83	0	0	0	0	0	0	17	0
IDMSS00057	154	0	82	0	0	0	0	0	0	18	0
IDMSS00104	147	0	0	79	0	0	0	0	21	0	0

In the most cases, a perfect monopoly is detected (81.4% of markets). A further 10.5% of markets is characterised by the presence, beside a very large agent with more than 90% of the market, of a sort of competitive fringe, accounting cumulatively from 1% to 9%. It is tempting to call these markets as "imperfect monopoly"[20]. They represent a sort of puzzle for the traditional models of industrial economics, since models based on costs would need to reconcile large economies of scale (necessary for the dominance of one agent) with the contemporary presence of very small agents (which in such setting should be at great cost disadvantage). None of the top 20 MSS has a symmetric structure: the agents have very different, even extremely polarised, market share.

As you can see in Appendix B, where you find the full results, the perfect duopoly is in the ranking position #55, with 55 occurrences. The symmetric oligopoly with three firms (IDMSS08988) can claim only 4 occurrences. There are no occurrences for IDMSS16247, which divide the market in 4 or 5 companies of equal size. But the most striking feature is that not a single occurrence is found for the market share structure that would contain perfectly competitive markets (IDMSS18429). Even relaxing the conditions and including neighbouring IDMSS does not allow finding any single instance. The most competitive market found (with the lowest Herfindahl index as computed according to the rules laid down in Appendix C) is IDMSS18160, with 22 in x_8, 43 in x_9 and 35 in x_{10}. To repeat, perfect competition has not been detected in any of the 98 269 markets, referring to 128 different countries[21]. If you are used to models that assume that all markets are perfectly competitive, think again: these models are completely at odds with the detected reality.

Research agenda

We first indicate possible new undertakings starting from this specific empirical analysis and then we open to alternative datasets and empirical strategies.

A first future work operating on the results for this particular, yet significant, database of market share structure derives from the finding that, even if the database contains several times the total number of MSS of the exhaustive list, only 1 008 out of 18 429 actually have been empirically detected (the 6%). 94% of theoretically possible elements of the morphospace of market shares do not exist and, as McGee (2006) would say, it means that there may be evolutionary forces preventing them to come into existence. A very interesting topics would be to characterise the existing from the non-existing market share structures and find a compact, maybe multi-layered and hierarchical, characterisation. It might be obtained by applying the technique of "classification trees" to the list we found, by separating two category (detected and non-detected), choosing a cut-off threshold and running the suitable statistical routines (as they available in statistical professional software).

The second future work could be to explore the reasons for a specific MSS to emerge, first in static terms and then in dynamic (evolutionary) terms. In particular, one might want to generate the empirical transition matrix that attribute the frequence with which a certain MSS gives way to another MSS in the next period. This is a typical Markov-process analysis applied to vectors. Please note, however, that evolution may require a non-Markov process in which also previous states and bifurcations do contribute to the trajectory that MSS take.

In this search for reasons of the MSS and their sequence over time, economists will probably draw on cost structures, on the one hand, and on product differentiation and consumers' taste heterogeneity,

[20] This term was introduced by Forchheimer (1908) to cope with the situation in which "one large dominant firm exists alongside a host of smaller companies on the so-called competitive fringe" (introductory remarks by R. Peterson to Forschheimer and Kuhn 1983).

[21] This is not to mean that there are always only few agents on the markets: this number can reach as high as 81. Simply their market share is such that one or more agents are significantly larger than the others are.

on the other. The first type of explanation, e.g. that companies with lower cost end up with larger market shares, will encounter two issues: to explain why on the market there are so widely different cost structures (which might point to tacit and cumulative knowledge, since they are difficult to justify with universal and free knowledge) and how it is possible that an agent with a cost advantage in one market loses it in another one.

We are rather inclined to the second group of explanation, referring to the product differentiation and consumers' taste heterogeneity, but we also would like to insert a third possible venue for explanation: the heterogeneity of production composition of multiproduct firms.

A third future work can go below the level of the IDMSS and look for possible internal differences. For instance out of the many monopoly structures detected (IDMS00001), one can distinguish which ones correspond to total lack of imports (thus a domestic monopoly) from those that correspond to a total lack of domestic production, with imports coming from one country only. For this, you simply need the data in Appendix B.

Fourthly, the stability of the IDMSS is a relevant subject of enquiry. For instance, the 55 occurrences of the symmetric perfect duopoly happen in 41 country (out of 128), in 21 sectors (out of 28), in 17 years (out of 27, which conversely means that for 10 years they were not detected anywhere) and in no country for more than 2 contiguous years (for which the only example is in the Turks and Caicos Islands, in the sector of "Pottery china earthenware", between 1993 and 1994). Thus, the symmetric perfect duopoly is not only a far from universal feature, but it very unstable, with stability here having the the simple meaning of lasting for several years. In a more advanced meaning of stability as the capability of the market share structure to come back after a shock temporarily disrupts it, there are no instances of the perfect duopoly being reinstated after being disrupted. We call these two meanings of "stability" as "permanence" and "resilience". You can begin exploring the issue of which IDMSS have the highest stability by looking at Appendix B.

Taking seriously the international trade core of this dataset, a fifth line of work could be to map the IDMSS to the world system trichotomy (core, periphery, and semi-periphery) and the finer categorisation proposed by Piana (2006), building on an exhaustive catalogue of the quality of bilateral relationship. In Appendix B, you also find data for testing international gravity model approaches. Insisting on considering countries as companies, their multiproduct competition, if enhanced by price indexes, could lead to characterise the competitor-by-competitor relationship along the exhaustive taxonomy put forth by Piana (2014).

A sixth line of work could aggregate MSS over 2 or more product categories as well as over 2 or more countries, up to the closed global market. For this, you need the absolute size of each market, so that in the aggregation you give more weight to the larger market[22]. The interest for this undertaken is enhanced if viewed as an implementation of the topics of meso-shapes, considered as aggregation of micro-shapes and relevant building blocks of macro-shapes (Piana et al. 2020). This line of analysis could end up suggesting loose auto-similarity at different levels of the MSS hierarchical structure, recalling a typical feature of fractals, and promote some method to cope with the issue of splitting an aggregate into market shares (of smaller product categories or sub-national regions).

[22] You find it in column 237 of the file distributed from Appendix B.

This is some of the work that can directly build upon the database analysis we carried out[23]. But another strand of research can well go into applying the exhaustive list to other markets, possibly of actual brands in competition without country proxies as we were compelled to do. The most immediate task is to apply the exhaustive list to well-defined markets for which you have actual companies in competition, possibly with long time series.

For instance, as a handful of references in random order, you might want to analyse:

* industrial datasets such as the Bureau van Dijk's AMADEUS[24] or BACH and ERICA[25];

* Nielsen data about sales and brands[26];

* the yearly publication on the Market Share Reporter by Gale Research[27];

* the on-going data collection called "Profit impact of marketing strategy", currently operated by PIMS Associates[28];

* car sales and registrations, by country, model, and brand[29];

* the market shares of Internet browser, aggregating by families or down to the level of browser version[30].

By such analysis, you gain not only a clearer view of what happens but also you can shed light on (or even "discover") IDMSSS that we did not find in our own empirical example. You may detect a IDMSS beating our "most competitive" case (IDMSS18160).

More ambitiously, some researcher might want to completely supersede our work with an equally large number of market shares of actual companies instead of country proxies. She or he would need to have access to good data and the permission to publish a synthesis, if not the full dataset.

Finally, the empirical dynamics across IDMSS could be explored under a potential impulse-response framework where a sequence of share structures in advertising (sometimes called "share of voice") and competitiveness indicators (including e.g. mindshare[31]) interact with the IDMSS.

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[23] The International Trade and Production Database for Estimation (ITPD-E) is a further source for a large-scale analysis of domestic markets where countries could be used as proxies for companies. It can be downloaded from https://www.usitc.gov/data/gravity/itpde.htm.

[24] https://www.bvdinfo.com/

[25] https://www.bach.banque-france.fr/?lang=en

[26] https://www.nielsen.com/us/en/solutions/measurement/retail-measurement/

[27] https://www.gale.com/ebooks/9781410327277/market-share-reporter

[28] https://www.pimsassociates.com/strategic-benchmarking/

[29] E.g. https://www.acea.be/statistics/tag/category/by-manufacturer-registrations and https://www.statista.com/markets/419/topic/487/vehicles-road-traffic

[30] See e.g. https://gs.statcounter.com/browser-market-share

[31] See Gill (2013).

Conclusions

We provided a final and exhaustive list of market share structures, i.e. ways in which the totality of 100% can be split in an open-ended number of firms. We provided an identification number for each item of this list, calling for a partition to be established on the list, so that human-friendly new labels for all possible (qualitatively different) market share structures can be established. We classified with our list a very large number of real-world market share structures from an international database. We challenge the researchers' community to dig further in our results and to apply the list to other databases. We propose to modellers' community to check which models can generate the entire (or part of the) list, so that a comparison across models from different research tradition can be accomplished. We call for new models capable of being superior to the existing ones, in terms of this benchmark. In short, we laid down a research agenda stemming from the recognition that all possible market share structures have been enlisted and identified.

It's not worth repeating here the detailed suggestions that we included in the text. We rather would like to devote the final part of this paper to a quick look on the origins of our approach. This piece of research is embedded in a pluralistic vision of economics. Our pluralism is not eclecticism, an unsystematic and opportunistic alternation in the practical utilization of mutually incompatible research traditions. We strive to establish a broad foundation of description, on which different models can shed (partial or full) light. We believe in the cumulativeness of scientific progress within a certain tradition of thought and in the comparability of certain, inevitably superficial, results stemming from different traditions.

In the evolutionary tradition of thought we belong to, we reject reductionism in favour of an organised plurality. Instead of relying on a single variable to describe the market share structure, be it the variance of market shares or the Herfindahl index, we propose a full list of market share structures to be evaluated in their emergence and evolution.

Appendix A

The exhaustive list of market share structures (MSS) and their individual identification code (IDMSS) is here:

Excel (xlsx) format

Appendix B

The number of occurrences in the TradeProd database of the market share structures is in the first sheet of the file, whereas in the second sheet you analytically find the IDMSS and the percentages of the market structure per country, year, and ISIC code. In column IC you find the market size (in 000s of US Dollars). In column ID the number of countries selling on that market, including the country itself, if the case. In the third sheet, you find the expanded text for the ISIC code. In the fourth sheet, you find the country code, name and a few, mainly geographical, data.

Excel (xlsx) format

The market size is obtained by adjusting all export values to eliminate re-exports, following the methodology used in Ardelean et al. (2017): exports are assumed composed by domestic and imported flows, with a given unit of good *g*'s exports considered as a re-export with probability equal to the share of imports of good *g* in the total availability of good *g* in the country. Markets with a size smaller than 1000 USD have been deleted.

Appendix C

In this appendix we discuss and release a reference value for the Herfindahl index (short for Herfindahl – Hirschman index) for all the market share structures of the exhaustive list. We provide a theoretical computation under certain assumptions, which can be used to calculate further concentration indexes – such as C(5), the sum of the market share of the largest five companies. We also provide the empirical values of H in the dataset discussed in Part II of the paper. They are typically spread over a certain interval, for which we give a theoretical reference based on assumptions about how many firms share the same column.

The Herfindahl index (H) is one of the most commonly utilised single number indicator of the degree of concentration of the market, a key metrics deriving from market share structures. Its formula is the summation of the power of two of the market shares, expressed in integers [0,100] as we do in this paper. Its values ranges thus in [0, 10000].

For many IDMSS, the Herfindahl index is not univocal bur requires three additional assumptions, leading to what can be considered a reference value:

1. We obtain the number of companies that contributed to the sum in a column of the MSS vector by taking the integer of the division of the sum with the minimum threshold of the column. For instance a value of 66 in the column where companies have between 20% and 30% of the market leads to Int(66/20) = 3 companies.

2. We assume equal shares of such companies (in the example: 22% each).

3. We assume that the market share is at least 1%, in line with the general quantisation of the system. Under this assumption, a 9 in the last column of the MSS matrix corresponds to 9 companies each with the 1% of the market.

The first two assumptions tend to "underestimate" H and the third to "overestimate" H with respect to alternative assumptions.

We release such theoretical values, as well as the empirical value from the TradeProd database, in the following formats:

Excel (xlsx) format

The theoretical value is available for all 18249 IDMSSS. . For the 1008 empirically detected IDMSS the minimum and maximum values computed from the data is reported. For the 17 421 IDMSS which have not been detected in the empirical dataset the text "N.D." is inserted. In the 31% of cases the theoretical value is larger than the empirical minimum and lower that the empirical maximum. In about the 69% the theoretical value is outside the empirical interval. But the difference with the nearest empirical extreme is usually small, being lower than 5% in the 56% of cases and lower than 10% in the 94% of cases.

If, along with our research agenda, new large empirical datasets are interpreted by detecting IDMSS, this relationship between theoretical and empirical data will be refined. In particular, a new way to compute the theoretical reference (or a interval serving for the purpose) could be devised.

Appendix D

In this appendix, we outline the algorithm used to single out the IDMSS from any market share structure you might have (either from empirical data or from a model). We also distribute an Excel file in which you can simply copy & paste your data, click a button and get the IDMSS. However, if you need to repeat many times this procedure from a model, you may want to embed a module singling out the IDMSS directly in your language or set-up.

Excel file

The simplest use of the Excel file is by copying and pasting your data about markets in the following way. You compute for each firm present on the market its market share, expressed in decimal numbers in the interval [0,1]. Make sure that the total sum of market share is exactly 1.

You copy and paste such values in columns from M on, depending on how many firms you have. You take this number and write it in cell B1. If you have only one market, then write 1 in the cell B1. You click on the button with the caption "Detect the ID of the market share structures". The execution of the code will write in cell A5 the IDMSS and in the subsequent columns (B-K) the defining values of the IDMSS.

If you have more than one market (or you have several time periods for the same market) you simply fill more rows. Please insert in B1 the number of rows and in B2 the maximum number of firms in any market. If you fail to do so, the sum will not be extending to all firms, so the result will be incorrect.

The Excel file contains a macro so obviously you need to have allowed macros when prompted. In order to see the code, you need to click on Creation / editing mode button and double click on the button itself. In order to visualize the Creation / editing mode button, you need to click on Excel File menu, line Options. Among the Options, select "Personalize the ribbon". In the tab "Main Tabs" you need to select the option "Developer mode". Depending on the language and the version of your Excel you might need to carry out a slightly different procedure.

For your convenience, we report the full commented code here, written in Visual Basic for Applications:

```
' This sections reduces the time of computation.

Application.ScreenUpdating = False

EventState = Application.EnableEvents

Application.EnableEvents = False

CalcState = Application.Calculation

Application.Calculation = xlCalculationManual

PageBreakState = ActiveSheet.DisplayPageBreaks

ActiveSheet.DisplayPageBreaks = False


Dim vv(10) ' The ten members of the MSS vectors.

' vv(1) is defined as the sum of the shares of those firms whose market share is 90% or higher.

' vv(2) is defined as the sum of the shares of those firms whose market share is 80% or higher but lower than 90%.


' This sections fills the columns B-K with the values, for each market, of the ten members of the MSS vectors.

For i = 5 To 4 + Cells(1, 2).Value ' Loop covering all markets from 1 to the user-defined number of markets.

' The user sets the value by changing the Excel cell B1, which is called in the code cells(1,2) with 1 being the row and 2 the column.
```

```
totmarket = 0 ' checksum

   For j = 13 To 12 + Cells(2, 2).Value ' for all firms from 1 to the user-defined number of maximum number of firms.
   'The maximum is taken from the market where the number of firms is the highest.

      If Cells(i, j).Value > 0 Then ' If the market share of the firm j on the market i is not zero.
         v = 10 - Int(Cells(i, j) * 10): If v = 0 Then v = 1 ' It identifies in which item of the MSS vector the share of that company must be added.
         vv(v) = vv(v) + Round(Cells(i, j) * 100) ' It round the market share according to the banking convention and assures that it is expressed by a number between 0 and 100.
      End If
   Next j

   For vvv = 1 To 10
   Cells(i, vvv + 1) = vv(vvv) ' It fills the right cell with the value of the member
   totmarket = totmarket + vv(vvv)
   vv(vvv) = 0 ' It resets the variable so that it can be used for the next market
   Next vvv


'This sub-section makes sure that the sum of all market shares is 100
If totmarket <> 100 Then
   'It singles out which is the highest value among the members of the MSS vector, so to modify by 1 in case of discrepancies (sum of shares equal to 99 or 101)

   mmax = 0
   vmax = 0
   For jj = 1 To 10
   If Cells(i, 1 + jj) > mmax Then
   mmax = Cells(i, 1 + jj)
   vmax = jj
   End If
   Next jj

   If totmarket = 99 Then
   Cells(i, 1 + vmax) = mmax + 1

      Else ' it means that it's 101.
   Cells(i, 1 + vmax) = mmax - 1
```

```
   End If
End If

' This sections look through the Sheet4, where the definitions of IDMSS are given, to single out
' which IDMSS characterises each market.
done = 0
If Cells(i, 6).Value > 1 Then ' the feature of the first table in the sheet on separate columns for fast retrieval.

   ' modify from now on by adding one check, in accordance to the new master
   ' the references to this sheet (Main sheet) are already ok.

   For ii = 3 To 2404
   If Cells(i, 2) = Feuil4.Cells(ii, 2) And Cells(i, 3) = Feuil4.Cells(ii, 3) And Cells(i, 4) = Feuil4.Cells(ii, 4) And Cells(i, 5) = Feuil4.Cells(ii, 5) And Cells(i, 6) = Feuil4.Cells(ii, 6) And Cells(i, 7) = Feuil4.Cells(ii, 7) And Cells(i, 8) = Feuil4.Cells(ii, 8) And Cells(i, 9) = Feuil4.Cells(ii, 9) And Cells(i, 10) = Feuil4.Cells(ii, 10) And Cells(i, 11) = Feuil4.Cells(ii, 11) Then
   Cells(i, 1) = Feuil4.Cells(ii, 1) ' Fill in the IDMSS
   done = 1
   Exit For
   End If

   Next ii

   Else
      If Cells(i, 9) > 25 Then
      For ii = 3 To 6841
             If Cells(i, 2) = Feuil4.Cells(ii, 15) And Cells(i, 3) = Feuil4.Cells(ii, 16) And Cells(i, 4) = Feuil4.Cells(ii, 17) And Cells(i, 5) = Feuil4.Cells(ii, 18) And Cells(i, 6) = Feuil4.Cells(ii, 19) And Cells(i, 7) = Feuil4.Cells(ii, 20) And Cells(i, 8) = Feuil4.Cells(ii, 21) And Cells(i, 9) = Feuil4.Cells(ii, 22) And Cells(i, 10) = Feuil4.Cells(ii, 23) And Cells(i, 11) = Feuil4.Cells(ii, 24) Then
            Cells(i, 1) = Feuil4.Cells(ii, 14)
            done = 1
            Exit For
            End If
      Next ii
   End If
End If
If done = 0 Then
      If Cells(i, 11) > 10 Then
           For ii = 3 To 4701
```

```
            If Cells(i, 2) = Feuil4.Cells(ii, 28) And Cells(i, 3) = Feuil4.Cells(ii, 29) And Cells(i, 4) = Feuil4.Cells(ii, 30) And Cells(i, 5) = Feuil4.Cells(ii, 31) And Cells(i, 6) = Feuil4.Cells(ii, 32) And Cells(i, 7) = Feuil4.Cells(ii, 33) And Cells(i, 8) = Feuil4.Cells(ii, 34) And Cells(i, 9) = Feuil4.Cells(ii, 35) And Cells(i, 10) = Feuil4.Cells(ii, 36) And Cells(i, 11) = Feuil4.Cells(ii, 37) Then

            Cells(i, 1) = Feuil4.Cells(ii, 27)

            done = 1

            Exit For

            End If

        Next ii


      Else ' v10 less the 10

      For ii = 3 To 4491

            If Cells(i, 2) = Feuil4.Cells(ii, 41) And Cells(i, 3) = Feuil4.Cells(ii, 42) And Cells(i, 4) = Feuil4.Cells(ii, 43) And Cells(i, 5) = Feuil4.Cells(ii, 44) And Cells(i, 6) = Feuil4.Cells(ii, 45) And Cells(i, 7) = Feuil4.Cells(ii, 46) And Cells(i, 8) = Feuil4.Cells(ii, 47) And Cells(i, 9) = Feuil4.Cells(ii, 48) And Cells(i, 10) = Feuil4.Cells(ii, 49) And Cells(i, 11) = Feuil4.Cells(ii, 50) Then

           Cells(i, 1) = Feuil4.Cells(ii, 40)


            Exit For

            End If

        Next ii

    End If

End If


Next i ' It closes the loop opened by the For.


' This section restores the default values that were modified to reduce the time of computation

ActiveSheet.DisplayPageBreaks = PageBreakState

Application.Calculation = CalcState

Application.EnableEvents = EventState

Application.ScreenUpdating = True

Application.Calculation = xlCalculationAutomatic
```

Conflict of interest declaration

The author states that there is no conflict of interest.

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